1025

M.Sc. IV Semester Examination 2020

Subject : Mathematics

Paper : IV – Special Functions

Max.Marks : 85 Min.Marks: 29

Note: Attempt all questions.

Section –'A'
(Short Answer Type Questions)

1. Attempt any five parts. 5x5=25

(i) Define Beta functions.

(ii) Evaluate: $\int_0^\infty e^{-x^2}\,dx$

(iii) Write hyper geometric differential equations.

(iv) Explain generalized hyper geometric function.

(v) Show that : $\frac{d}{dx}(x^{-n}J_n(x)) = -x^{-n}\,J_{n+1}(x)$.

(vi) Evaluate $P_n(0)$

(vii) Prove that recurrence relation:

$$(2n+1)\,xP_n = (n+1)P_{n+1} + nP_{n-1}$$

(viii) Define Hermite polynomial.

(ix) Define Meijer's G-function.

(x) Define differential equation for G-function.

Section –'B'
(Long Answer Type Questions)

2. Prove that: $B(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ 12

OR

State and prove Gaus's multiplication theorem.

Contd…

3. If Re(c-a-b)>0, Re(c)>Re(b)>0 and c is neither zero nor a negative integer, then prove that:

$$F(a,b,c,1) = \frac{\Gamma c\ \Gamma(c-a-b)}{\Gamma(c-b)\ \Gamma(c-a)}$$ 12

OR

State and prove Dixon's theorem.

4. Prove that: 12

$$P_n(x) = \frac{1}{2^n!n} \frac{d^n}{dx^n} (x^2-1)^n$$

OR

Prove that the recurrence relation:

$$2nJ_n(z) = z[J_{n-1}(z) + J_{n+1}(z)]$$

5. Prove that: 12

$$H_n(x) = (-1)^n e^{x2} \frac{d^n}{dx^n} (e^{-x2})$$

OR

For Laguerre polynomials prove that:

$$\int_0^\infty e^{-x} L_m(x) L_n(x)\, dx = \begin{cases} 1 & if\ m = n \\ 0 & if\ m \neq n \end{cases}$$

6. State and prove simple multiplication theorem. 12

OR

Prove that:

$$\int_0^1 y^{\alpha-\beta-1} G_{pq}^{mn}(\gamma^2 \,|_{bs}^{ar})dy = \Gamma(\alpha-\beta)\ G_{p+1,q+1}^{m,n+1}\left(\times \,\middle|\, {\alpha,a1,a2,....ap \atop b1,b2,.....bp,\beta}\right)$$

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