1022

M.Sc. IV Sem. Main Examination 2020

MATHEMATICS

Paper - I

Partial Differential Equations

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M.M.: 85
Mini.Pass.Marks : 29

Note : Attempt all questions.

Q. 1 Attempt any five questions form the following. $5\times5=25$

(i) Derive the formation of partial differential equation of first order.

(ii) Form the PDE by eliminating the arbitrary function from -

$$f(x+y+z,\ x^2+y^2+z^2) = 0$$

(iii) Decide whether the following equation is hyperbolic, elliptic or parabolic -

$$2\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

Contd..

(iv) Classify the PDE -

$$x^2 u_{xx} - 2xy\, u_{xy} + y^2 u_{yy} = e^x$$

Also, find the characteristics equation.

(v) Derive the Laplace equation.

(vi) Derive the Poison's equation.

(vii) Define Dirac-Delta function.

(viii) Write the elementry solution of the one dimensional diffusion equation.

(ix) Reduce the one dimensional wave equation

$u_{tt} - c^2 u_{xx} = 0$, to canonical form.

(x) Define D'Alemberts solution for one dimensional wave equation.

UNIT - I

Q.2 Find the integral surface of the linear PDE - 12

$$xp + yq = z$$

which contains the circle defined by

$$x^2+y^2+z^2 = 4, \quad x+y+z = 2$$

Or

Find the complete integral of PDE -

$$z^2 = pq\,xy$$

by charpit method.

UNIT - II

Q.3 Reduce the following equation to canonical form - 12

$$u_{xx} + x\,u_{yy} = 0, \quad x \neq 0$$

Or

Reduce the following second order partial differential equation to canonical form and hence solve it,

$$y\,u_{xx} + (x+y)\,u_{xy} + x\,u_{yy} = 0, \quad x \neq 0, \quad y \neq 0$$

UNIT - III

Q.4 Solve the following Dirichlet problem for a ractangle : 12

$$\nabla^2 u = 0, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b$$

B.CS : $u(x,b) = u(a,y) = 0$

$$u(0,y) = 0, \qquad u(x,0) = f(x)$$

Contd..

Or

Discuss the solution of Laplace equation in cylindrical coordinates.

UNIT - IV

Q.5 Solve the following initial boundary value problem of heat conduction which is given by - 12

PDE : $\dfrac{\partial U}{\partial t} = a \dfrac{\partial^2 U}{\partial x^2}$, $0 \le x \le L, \quad t > 0$

B.CS : $\partial U(0,t) = 0, \quad t \ge 0$

$\dfrac{\partial U}{\partial x}(L,t) = 0, \quad t \ge 0$

I.C : $U(x,0) = U_0, \quad 0 \le x \le L$

Or

The ends A and B of a rod, 10 cm is length, are kept at temperatures $0°C$ and $100°C$ until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20°C, and the end B is decreased to 60°C. Find the temperature distribution in the rod at time ***t***.

Contd..

UNIT - V

Q.6 Obtain the solution of the following wave equation with prescribed boundary and initial conditions : 12

PDE : $u_{tt} - c^2 u_{xx} = 0, \quad 0 \le x \le L, \; t > 0$

B.CS : $u(0,t) = 0, \quad t > 0$

$u(L,t) = 0, \; t > 0$

I.CS : $u(x,0) = f(x), \; 0 \le x \le L$

$u_t(x,0) = g(x)$

Or

Solve the following wave equation with prescribed boundary and initial conditions :

PDE : $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = F(x,t), \quad 0 \le x \le L, \quad t \ge 0$

B.CS : $u(0,t) = u(L,t) = 0, \quad t \ge 0$

I.CS : $u(x,0) = u_t(x,0) = 0, \quad 0 \le x \le L$

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